

String inspired explanation for the super-acceleration of our universe

Ahmad Sheykhi ^{1,2,3*} and Bin Wang ^{1†}

¹ *Department of Physics, Fudan University, Shanghai 200433, China*

² *Department of Physics, Shahid Bahonar University, Kerman, Iran*

Nematollah Riazi [‡]

³*Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran*

We investigate the effect of the bulk content in the general Gauss-Bonnet braneworld on the evolution of the universe. We find that the Gauss-Bonnet term and the combination of the dark radiation and the matter content of the bulk play a crucial role in the universe evolution. We show that our model can describe the super-acceleration of our universe with the equation of state of the effective dark energy in agreement with observations.

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I. INTRODUCTION

One of the most dramatic discoveries of the modern cosmology in the past decade is that our universe is currently accelerating [1]. A component that causes the accelerated expansion of the universe is referred to as “dark energy”. Although the nature of such dark energy is still speculative, an overwhelming flood of papers has appeared which attempt to describe it by devising a great variety of models. Among them are cosmological constant, exotic fields such as phantom or quintessence, modified gravity, etc, see [2][3] for a recent review. Available models of dark energy differ in the value and variation of the equation of state parameter w during the evolution of the universe. The cosmological constant with $w = -1$, is located at a central position among dark energy models both in theoretical investigation and in data analysis [4]. In quintessence [5], Chaplygin gas [6] and holographic dark energy models [7], w always stays bigger than -1 . The phantom models of dark energy have $w < -1$ [8]. However, following the more accurate data analysis, a more dramatic result appears showing that the time varying dark energy gives a better fit than a cosmological constant and in particular, w can cross -1 around $z = 0.2$ from above to below [9]. Although the galaxy cluster gas mass fraction data do not support the time-varying w [10], theoretical attempts towards the understanding of the w crossing -1 phenomenon have been started. Some dark energy models, such as the one containing a negative kinetic scalar field and a normal scalar field [11], or a single scalar field model [12] and interacting holographic dark energy models [13] have been constructed to gain insight into the occurrence of the transition of the dark energy equation of state and the mechanism behind this transition. Other studies on the $w = -1$ crossing have been carried out in [14].

Independent of the challenge we deal with the dark energy puzzle, in recent years, theories of large extra dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have received a lot of interest. According to the braneworld scenario the standard model of particle fields are confined on the brane while, in contrast, the gravity is free to propagate in the whole spacetime. In these theories the cosmological evolution on the brane is described by an effective Friedmann equation that incorporates non-trivially with the effects of the bulk onto the brane. An interesting consequence of the braneworld scenario is that it allows the presence of five-dimensional matter which can propagate in the bulk space and may interact with the matter content in the braneworld. It has been shown that such an interaction can alter the profile of the cosmic expansion and lead to a behavior that would resemble the dark energy. The cosmic evolution of the braneworld with energy exchange between brane and bulk has been studied in different setups [15][16][17][18]. In these models, due to the energy exchange between the bulk and the brane, the usual energy conservation law on the brane is broken and consequently it was found that the equation of state of the effective dark energy may experience the transition behavior (see e.g [17, 18]).

On the other hand, in string theory, in addition to the Einstein action, some higher derivative curvature terms have been included to derive the gravity. In order to obtain a ghost-free theory, the combination of quadratic terms called Gauss-Bonnet term is usually employed as curvature corrections to the Einstein-Hilbert action [19]. From a geometric point of view, the combination of the Einstein-Hilbert and Gauss-Bonnet term constitutes, for 5D spacetimes, the

* asheykhi@mail.uk.ac.ir

† wangb@fudan.edu.cn

‡ riazi@physics.susc.ac.ir

most general Lagrangian to produce second-order field equations [20]. The Gauss-Bonnet correction significantly changes the bulk field equations and leads to modifications in the braneworld Friedmann equations. Therefore, the study of the effects of the Gauss-Bonnet correction term on the evolution of the universe in the braneworld scenario is well motivated. Influences of the Gauss-Bonnet correction on the DGP braneworld have been studied in [21, 22].

The purpose of the present work is to investigate the effects of the bulk content in the general Gauss-Bonnet braneworld on the evolution of the universe. Although the effects of the Gauss-Bonnet correction term on the late time universe is small, we will see that it still plays an important role in the cosmic evolution. Besides we will show that the combination of the dark radiation term and the matter content of the bulk plays the role of the dark energy on the brane and influences the evolution of the universe. In our model, in contrast to the previous models ([15, 16, 17, 18]), we do not need to break down the standard energy momentum conservation law on the brane, although our model can allow such assumption if one is interested. We will show that by suitably choosing model parameters, our model can exhibit accelerated expansion of the universe. In addition, we will present a profile of the w crossing -1 phenomenon which is in good agreement with observations.

The paper is organized as follows. In Section II, we present a braneworld model to describe the accelerated expansion and the effective equation of state of dark energy in the presence of the Gauss-Bonnet correction term in the bulk. In Section III, we study the cosmological consequences of the model and in particular, its effect on the evolution of the universe. The last section is devoted to conclusions and discussions.

II. THE MODEL

The theory we are considering is five-dimensional and has an action of the form

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} (R - 2\Lambda + \alpha \mathcal{L}_{GB}) + \int d^5x \sqrt{-g} \mathcal{L}_{bulk}^m + \int d^4x \sqrt{-\tilde{g}} (\mathcal{L}_{brane}^m - \sigma), \quad (1)$$

where $\Lambda < 0$ is the bulk cosmological constant and \mathcal{L}_{GB} is the Gauss-Bonnet correction term

$$\mathcal{L}_{GB} = R^2 - 4R^{AB}R_{AB} + R^{ABCD}R_{ABCD}. \quad (2)$$

Here g and \tilde{g} are the bulk and brane metrics, respectively. R , R_{AB} , and R_{ABCD} are the scalar curvature and Ricci and Riemann tensors, respectively. Throughout this paper we choose the unit so that $\kappa^2 = 1$ as the gravitational constant in five dimension. We have also included arbitrary matter content both in the bulk and on the brane through \mathcal{L}_{bulk}^m and \mathcal{L}_{brane}^m respectively, and σ is the positive brane tension. The field equations can be obtained by varying the action (1) with respect to the bulk metric g_{AB} . The result is

$$G_{AB} + \Lambda g_{AB} + 2\alpha H_{AB} = T_{AB}, \quad (3)$$

where H_{AB} is the second-order Lovelock tensor

$$H_{AB} = RR_{AB} - 2R_A^C R_{BC} - 2R^{CD} R_{ACBD} + R_A^{CDE} R_{BCDE} - \frac{1}{4} g_{AB} \mathcal{L}_{GB}.$$

For convenience and without loss of generality, we can choose the extra-dimensional coordinate y such that the brane is located at $y = 0$ and bulk has \mathbb{Z}_2 symmetry. We are interested in the cosmological solution with a metric

$$ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2, \quad (4)$$

where γ_{ij} is a maximally symmetric 3-dimensional metric for the surface ($t=\text{const.}$, $y=\text{const.}$), whose spatial curvature is parameterized by $k = -1, 0, 1$. The metric coefficients n and b are chosen so that, $n(t, 0) = 1$ and $b(t, 0) = 1$, where t is cosmic time on the brane. The total energy-momentum tensor has bulk and brane components and can be written as

$$T_{AB} = T_{AB}|_{brane} + T_{AB}|_{\sigma} + T_{AB}|_{bulk}. \quad (5)$$

The first and the second terms are the contribution from the energy-momentum tensor of the matter field confined to the brane and the brane tension

$$T_B^A|_{brane} = \text{diag}(-\rho, p, p, p, 0) \frac{\delta(y)}{b}, \quad (6)$$

$$T_B^A|_{\sigma} = \text{diag}(-\sigma, -\sigma, -\sigma, -\sigma, 0) \frac{\delta(y)}{b}, \quad (7)$$

where ρ , and p , being the energy density and pressure on the brane, respectively. In addition we assume an energy-momentum tensor for the bulk content of the form

$$T^A_B|_{bulk} = \begin{pmatrix} T^0_0 & 0 & T^0_5 \\ 0 & T^i_j \delta^i_j & 0 \\ -\frac{n^2}{b^2} T^0_5 & 0 & T^5_5 \end{pmatrix}. \quad (8)$$

The quantities which are of interest here are T^5_5 and T^0_5 , as these two enter the cosmological equations of motion. In fact, T^0_5 is the term responsible for energy exchange between the brane and the bulk. Integrating the (00) component of the field equations (3) across the brane and imposing \mathbb{Z}_2 symmetry, we have the jump across the brane [23]

$$\left[1 + 4\alpha \left(H^2 + \frac{k}{a_0^2} - \frac{a'^2_+}{3a_0^2} \right) \right] \frac{a'_+}{a_0} = -\frac{1}{6}(\rho + \sigma), \quad (9)$$

where $2a'_+ = -2a'_-$ is the discontinuity of the first derivative. $H = \dot{a}_0/a_0$ is the Hubble parameter on the brane. Eq. (9) is a cubic equation for the discontinuity a'_+/a_0 , which has only one real solution, the other two being complex. Therefore, if we require our cosmological equations to have the right $\alpha \rightarrow 0$ limit we are left with only one solution. However, this real root is too lengthy and complicated to present here. Since we are interested to study the effect of the Gauss-Bonnet correction term on the evolution of the universe in the late time so it is reasonable to choose the Gauss-Bonnet coupling constant α to be small, namely $0 < \alpha < 1$. Using this fact we can expand the real solution for a'_+/a_0 versus α powers. The result for $k = 0$ up to order α is

$$\frac{a'_+}{a_0} = -\frac{1}{6}(\rho + \sigma) + \frac{\alpha}{162} (\rho + \sigma) \left(108 H^2 - (\rho + \sigma)^2 \right) + O(\alpha^2). \quad (10)$$

In a similar way, integrating the (ij) component of the field equations (3) across the brane and imposing \mathbb{Z}_2 symmetry, we can obtain the discontinuity in the metric function n'_+/n_0 , which for $k = 0$ can be written up to $O(\alpha)$ in the following form

$$\begin{aligned} \frac{n'_+}{n_0} = & \frac{1}{6}(2\rho + 3p - \sigma) + \frac{\alpha}{3} \left(-2 H^2 (2\rho + 3p - \sigma) \right. \\ & \left. + \frac{1}{54} (\rho + \sigma)^2 (8\rho + 9p - \sigma) + 4\dot{H} (\rho + \sigma) \right) + O(\alpha^2), \end{aligned} \quad (11)$$

where dots denote time derivatives and primes denote derivatives with respect to y . At this point we find it convenient to absorb the brane tension σ in ρ and p with the replacement $\rho + \sigma \rightarrow \rho$ and $p - \sigma \rightarrow p$. Therefore the junction conditions (10) and (11) can be simplified

$$\frac{a'_+}{a_0} = -\frac{\rho}{6} + \frac{\alpha}{162} \rho (108 H^2 - \rho^2), \quad (12)$$

$$\frac{n'_+}{n_0} = \frac{1}{6}(2\rho + 3p) + \frac{\alpha}{3} \left(-2 H^2 (2\rho + 3p) + \frac{\rho^2}{54} (8\rho + 9p) + 4\dot{H}\rho \right). \quad (13)$$

Substituting the junction conditions (12) and (13) into the (55) and (05) components of the field equations (3), we obtain the modified Friedmann equation and the semi-conservation law on the brane (up to order α)

$$\begin{aligned} H^2 \left(1 - \frac{\alpha}{9} \rho (2\rho + 3p) \right) + \left(\dot{H} + H^2 \right) \left(1 + 4\alpha \left(H^2 + \frac{\rho^2}{36} \right) \right) \\ + \frac{\rho}{36} (\rho + 3p) + \frac{\alpha}{972} \rho^3 (2\rho + 3p) = \frac{\Lambda - T^5_5}{3}, \end{aligned} \quad (14)$$

and

$$\dot{\rho} + 3H(\rho + p) = -T, \quad T \equiv 2T^0_5 \left[1 - 4\alpha \left(H^2 - \frac{\rho^2}{36} \right) \right]. \quad (15)$$

We shall assume an equation of state $p = w\rho$ to hold between the energy density and pressure of matter on the brane. Therefore we have

$$\begin{aligned} H^2 \left(1 - \frac{\alpha}{9} \rho^2 (2 + 3w) \right) + \left(\dot{H} + H^2 \right) \left(1 + 4\alpha \left(H^2 + \frac{\rho^2}{36} \right) \right) \\ + \frac{\rho^2}{36} (1 + 3w) + \frac{\alpha}{972} \rho^4 (2 + 3w) = \frac{\Lambda - T^5_5}{3}, \end{aligned} \quad (16)$$

$$\dot{\rho} + 3H\rho(1 + w) = -T. \quad (17)$$

One can easily check that in the limit $\alpha \rightarrow 0$, Eqs. (10)-(17) reduce to the corresponding equations of the braneworld model without Gauss-Bonnet correction term [15].

Remarkably, we can show that the Friedmann equation (16) is equivalent to the following equations

$$2\alpha H^4 + \left(1 + \frac{\alpha\rho^2}{9}\right) H^2 = \frac{\rho^2}{36} \left(1 + \frac{\alpha\rho^2}{54}\right) + \chi + \frac{\Lambda}{6} - \frac{T_5^5}{3}, \quad (18)$$

with χ satisfying

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = \frac{2}{36} T \rho \left[1 - 4\alpha \left(H^2 - \frac{\rho^2}{108}\right)\right] + \frac{\dot{T}_5^5}{3}. \quad (19)$$

Using the definition for T in Eq. (15), the latter equation up to order α can be written as

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = \frac{4}{36} T_5^0 \rho \left[1 - 8\alpha \left(H^2 - \frac{\rho^2}{54}\right)\right] + \frac{\dot{T}_5^5}{3}. \quad (20)$$

Eq. (18) is the modified Friedmann equation describing cosmological evolution on the brane. The auxiliary field χ incorporates non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane. The bulk matter contributes to the energy content of the brane through the bulk pressure terms T_5^5 that appear in the right hand side of the Friedmann equation. In addition, the bulk matter contributes to the energy conservation equation (15) through T_5^0 which is responsible for the energy exchange between the brane and bulk. The functions T_5^5 and T_5^0 are functions of time corresponding to their values on the brane. The energy-momentum conservation $\nabla_A T_B^A = 0$ cannot fully determine T_5^5 and T_5^0 and a particular model of the bulk matter is required [18]. In the limit $\alpha \rightarrow 0$, Eqs. (18) and (20) reduce to (after replacement $\rho \rightarrow \rho + \sigma$)

$$H^2 = \frac{(\rho + \sigma)^2}{36} + \chi + \frac{\Lambda}{6} - \frac{T_5^5}{3}, \quad (21)$$

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = \frac{4}{36} T_5^0 (\rho + \sigma) + \frac{\dot{T}_5^5}{3}. \quad (22)$$

If we invoke the usual definition $\beta \equiv 1/36$, $\lambda \equiv (\Lambda + \sigma^2/6)/6$ and $\gamma \equiv \sigma\beta$, we get ($\kappa^2 = 1$)

$$H^2 = \beta\rho^2 + 2\gamma\rho + \lambda + \chi - \frac{T_5^5}{3},$$

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = 4T_5^0(\beta\rho + \gamma) + \frac{\dot{T}_5^5}{3}, \quad (23)$$

which is noting, but the general set of the equations in RS II braneworld model with bulk matter content plus brane-bulk energy exchange (see for example [18]).

Returning to the general Friedman equation (18) with Gauss-Bonnet correction term, we can show that this equation has the solution for H of the form

$$H^2 = -\frac{1}{4\alpha} - \frac{\rho^2}{36} \pm \frac{1}{108\alpha} [729 + 12\alpha\rho^2(27 + \alpha\rho^2) + 972\alpha(6\chi + \Lambda - 2T_5^5)]^{1/2}. \quad (24)$$

The upper solution (+) has correct $\alpha \rightarrow 0$ limit. Indeed, if we expand this solution verses α we get (up to $O(\alpha)$)

$$H^2 = \frac{\rho^2}{36} + \chi + \frac{\Lambda}{6} - \frac{T_5^5}{3} - \frac{\alpha}{18} \left[2\rho^2 \left(\frac{\rho^2}{27} + 2\chi + \frac{\Lambda}{3} - \frac{2T_5^5}{3}\right) + (6\chi + \Lambda - 2T_5^5)^2\right], \quad (25)$$

and Eqs. (15) and (20) become

$$\dot{\rho} + 3H\rho(1 + \omega) = -2T_5^0 \left[1 - 4\alpha \left(\chi + \frac{\Lambda}{6} - \frac{T_5^5}{3}\right)\right], \quad (26)$$

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = \frac{4}{36} T_5^0 \rho \left[1 - 8\alpha \left(\frac{\rho^2}{108} + \frac{\Lambda}{6} - \frac{T_5^5}{3} + \chi\right)\right] + \frac{\dot{T}_5^5}{3}. \quad (27)$$

Therefore, until now we have obtained the set of equations describing the dynamics of our universe (Eqs. (25)-(27)) in the general Gauss-Bonnet braneworld with both bulk matter content and bulk-brane energy exchange provided that the Gauss-Bonnet coupling constant α is chosen sufficiently small. It is worth noting that although α is small, it has a dramatic effect on the dynamic behavior of the cosmic evolution. Besides the appearance of the ρ^4 term on the right hand side of Eq. (25) shows that in high energy scale the Gauss-Bonnet correction term plays an important role.

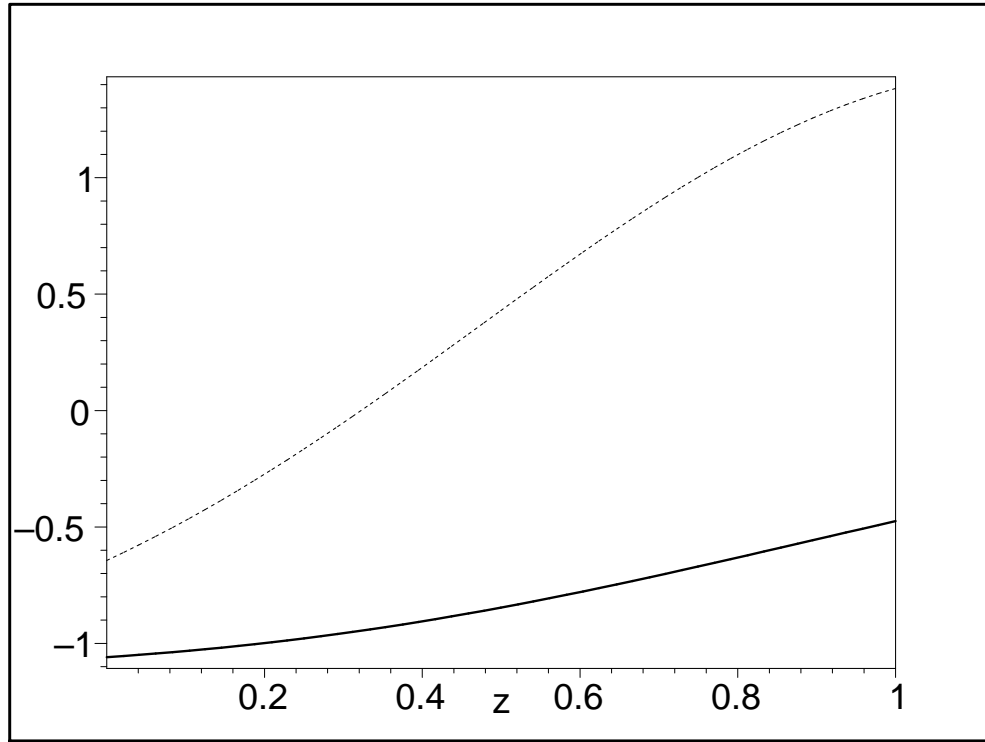


Figure 1: Evolution of $w_{\text{eff}}(z)$ (bold line) and $q(z)$ (dashed line) versus z for $\nu = 0.34$ and $\alpha = 0$.

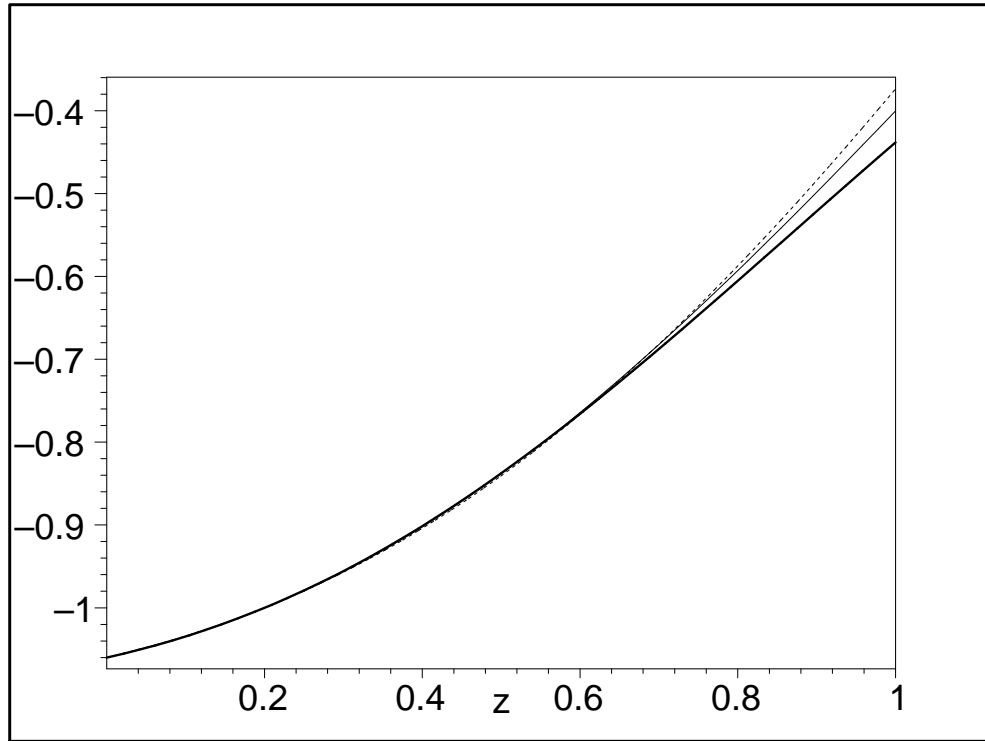


Figure 2: Evolution of $w_{\text{eff}}(z)$ versus z for $0 < \nu \leq 0.17$. $\alpha = 0.01$ (bold line), $\alpha = 0.1$ (continuous line), and $\alpha = 0.9$ (dashed line).

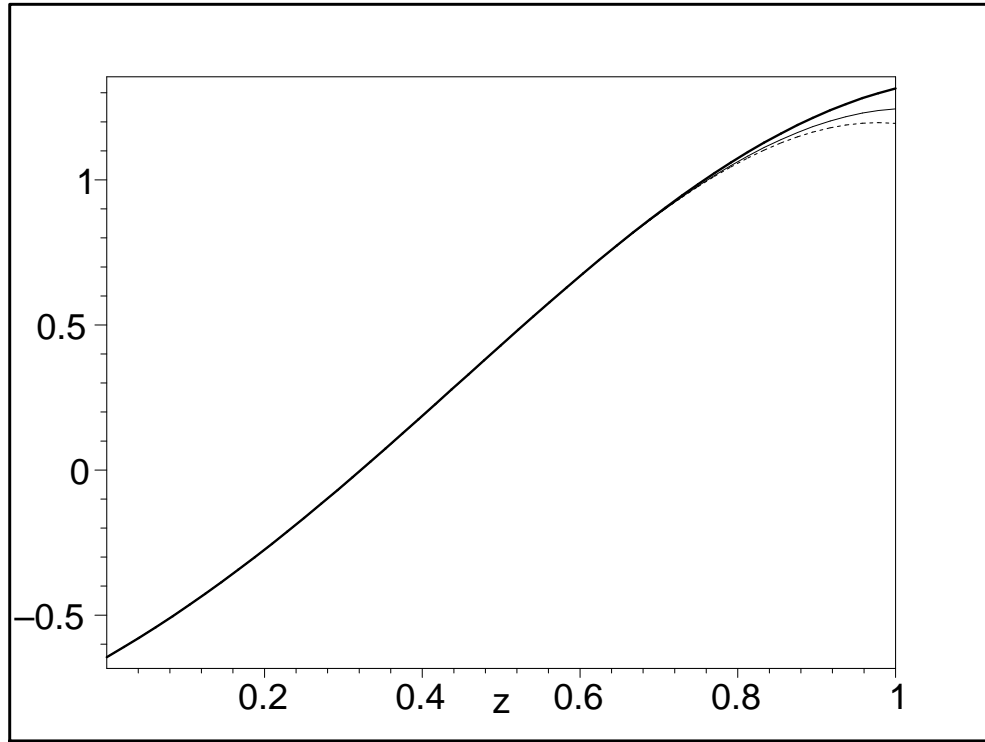


Figure 3: Evolution of $q(z)$ versus z for $0 < \nu \leq 0.17$. $\alpha = 0.01$ (bold line), $\alpha = 0.1$ (continuous line), and $\alpha = 0.9$ (dashed line).

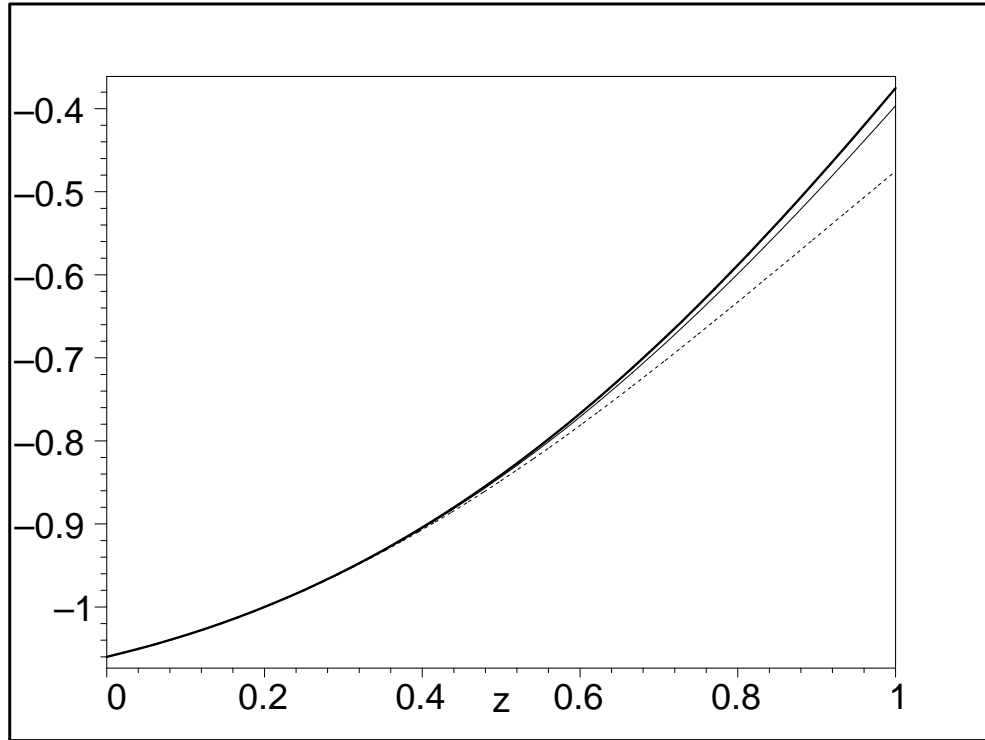


Figure 4: Evolution of $w_{\text{eff}}(z)$ versus z for $0.18 \leq \nu \leq 0.34$. $\alpha = 0.01$ (bold line), $\alpha = 0.1$ (continuous line), and $\alpha = 0.9$ (dashed lined).

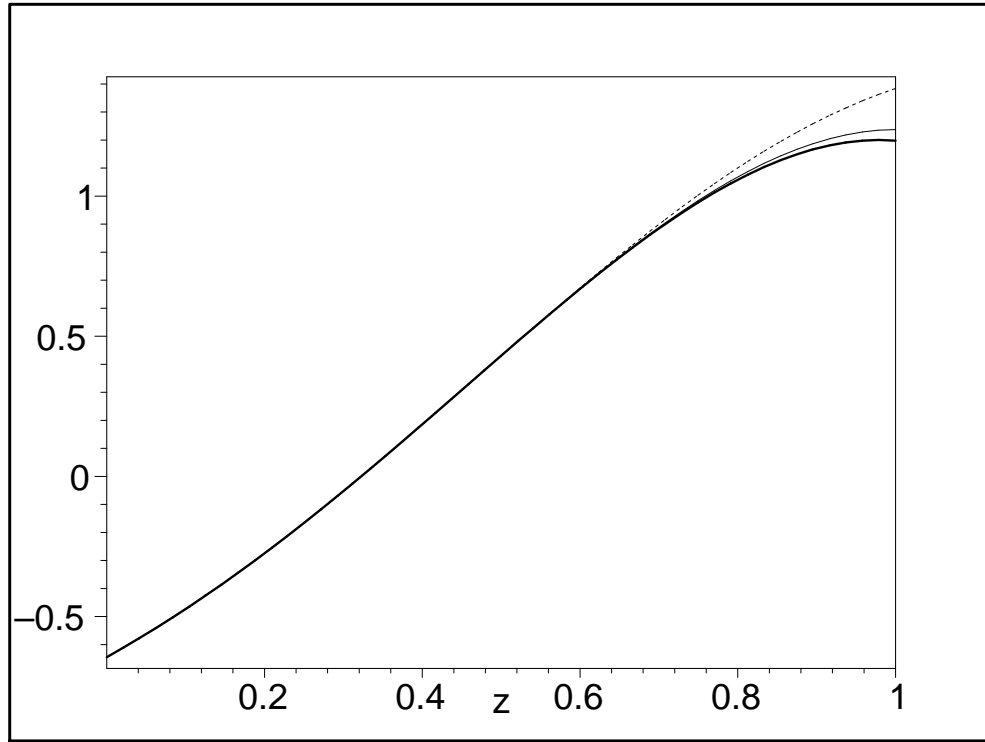


Figure 5: Evolution of $q(z)$ versus z for $0.18 \leq \nu \leq 0.34$. $\alpha = 0.01$ (bold line), $\alpha = 0.1$ (continuous line), and $\alpha = 0.9$ (dashed line).

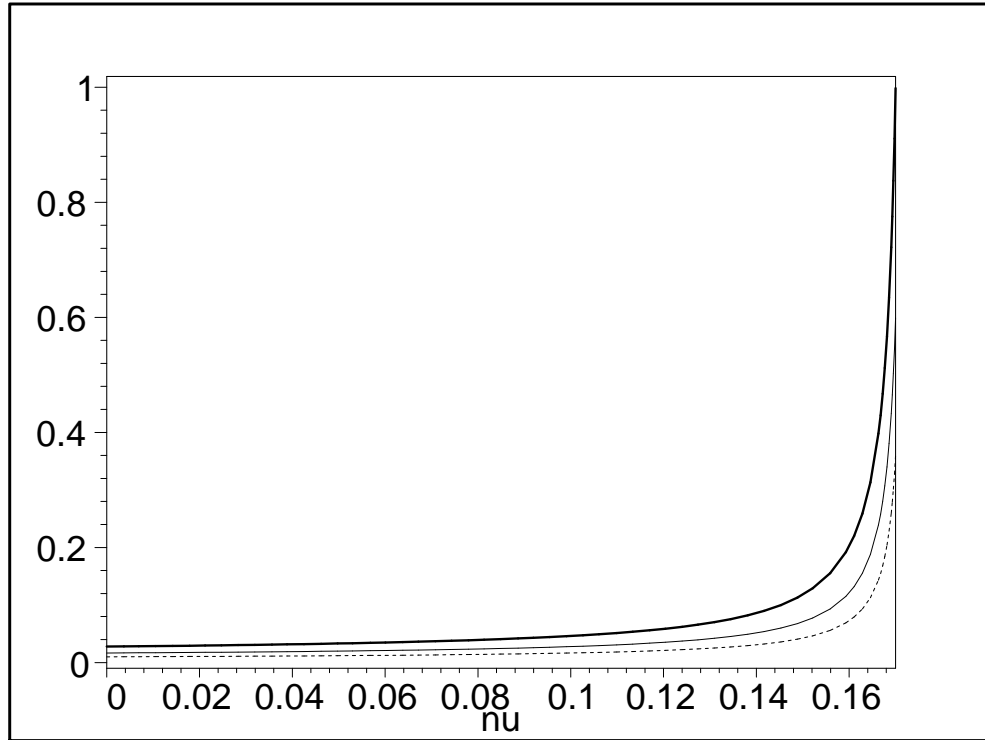


Figure 6: The parameter space of the function $\alpha(\nu, A)$ for $0 < \nu \leq 0.17$. $A = 18$ (bold line), $A = 30$ (continuous line), and $A = 50$ (dashed line).

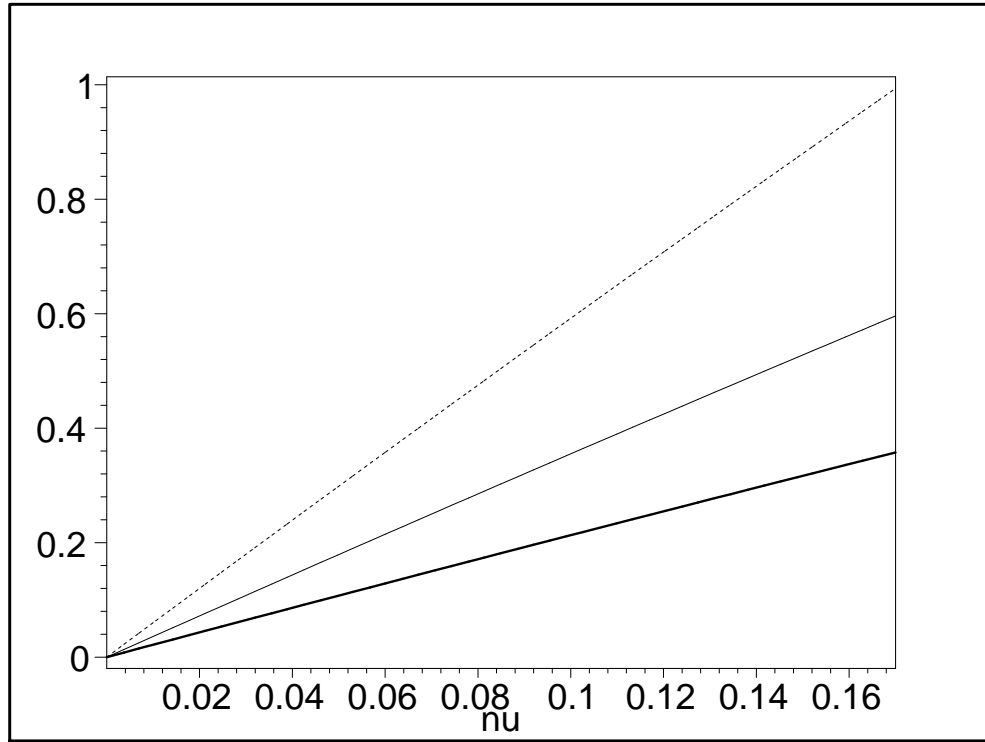


Figure 7: The parameter space of the function $C(\nu, A)$ for $0 < \nu \leq 0.17$. $A = 18$ (bold line), $A = 30$ (continuous line), and $A = 50$ (dashed line).

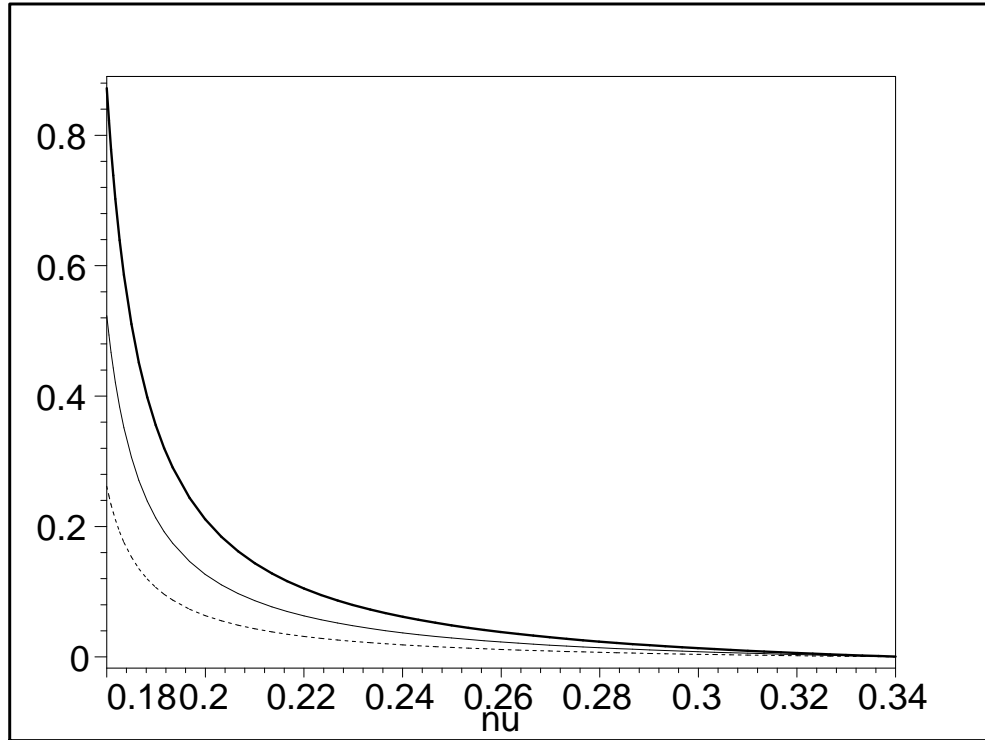


Figure 8: The parameter space of the function $\alpha(\nu, A)$ for $0.18 \leq \nu \leq 0.34$. $A = -6$ (bold line), $A = -10$ (continuous line), and $A = -20$ (dashed line).

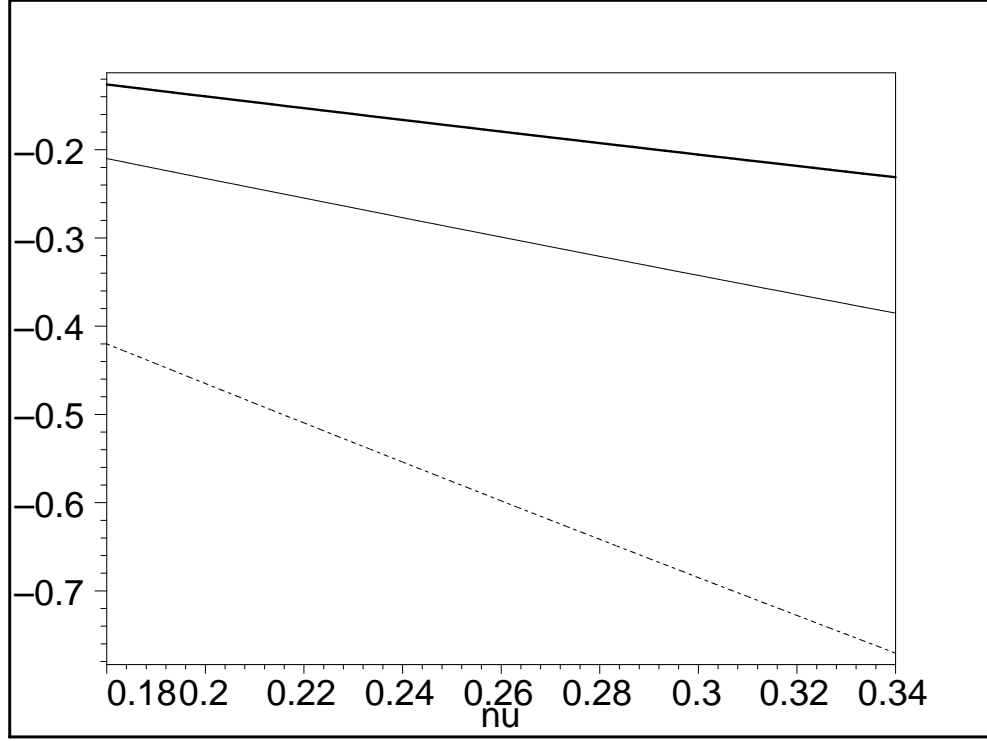


Figure 9: The parameter space of the function $C(\nu, A)$ for $0.18 \leq \nu \leq 0.34$. $A = -6$ (bold line), $A = -10$ (continuous line), and $A = -20$ (dashed line).

III. COSMOLOGICAL CONSEQUENCES

In this section we are going to explore some cosmological consequences of our model. To do this, first we separate back the matter energy density and the brane tension as usual form with the replacement $\rho \rightarrow \rho + \sigma$. Therefore Eqs. (25) and (27) become

$$H^2 = \frac{2\sigma\rho}{36} \left(1 + \frac{\rho}{2\sigma}\right) + \frac{1}{6} \left(\Lambda + \frac{\sigma^2}{6}\right) - \frac{T_5^5}{3} + \chi - \frac{\alpha}{18} \left[2\sigma^2 \left(1 + \frac{\rho}{\sigma}\right)^2 \left(\frac{\sigma^2}{27} \left(1 + \frac{\rho}{\sigma}\right)^2 + 2\chi + \frac{\Lambda}{3} - \frac{2T_5^5}{3}\right) + (6\chi + \Lambda - 2T_5^5)^2 \right], \quad (28)$$

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = \frac{4\sigma}{36} T_5^0 \left(1 + \frac{\rho}{\sigma}\right) \left[1 - 8\alpha \left(\frac{\sigma^2}{108} \left(1 + \frac{\rho}{\sigma}\right)^2 + \frac{\Lambda}{6} - \frac{T_5^5}{3} + \chi\right)\right] + \frac{\dot{T}_5^5}{3}. \quad (29)$$

We are interested in the scenarios where the energy density of the brane is much lower than the brane tension, namely $\rho \ll \sigma$. Assuming the Randall-Sundrum fine-tuning $\Lambda + \sigma^2/6 = 0$ holds on the brane and defining the parameter $\gamma \equiv \sigma/36$, Eqs. (28) and (29) can be simplified in the following form

$$H^2 = 2\gamma\rho + \chi - \frac{T_5^5}{3} - \frac{\alpha}{18} \left[2 \left(\chi - \frac{T_5^5}{3}\right) \left(\sigma^2 + 18 \left(\chi - \frac{T_5^5}{3}\right)\right) - \frac{\sigma^4}{108} \right], \quad (30)$$

$$\dot{\chi} + 4H \left(\chi - \frac{T_5^5}{6}\right) = 4\gamma T_5^0 \left[1 - 8\alpha \left(\chi - \frac{T_5^5}{3} - \frac{\sigma^2}{54}\right)\right] + \frac{\dot{T}_5^5}{3}. \quad (31)$$

Now, one may adopt several strategies to find solutions of Eqs. (26), (30) and (31). For example, one may take a suitable ansatz for the time dependent functions T_5^0 and T_5^5 and using Eq. (31) to find the function χ . Then substitute χ , T_5^0 and T_5^5 into Eq. (26) one can try to obtain ρ , and finally one may find Hubble parameter H through

Eq. (30). In the following we are interested in the case in which the energy momentum conservation law on the brane holds, which is usually assumed in the braneworld scenarios. Indeed, we want to consider the effect of the bulk content on the evolution of the universe without brane-bulk energy exchange, therefore we set $T_5^0 = 0$. The case with brane-bulk energy exchange in the general Gauss-Bonnet braneworld will be addressed elsewhere. It was argued that the energy exchange between the bulk and brane T_5^0 will lead to the effective dark energy equation of state crossing -1 [16,17]. Here we will show that without the energy exchange, the effect of T_5^5 and the combined T_5^5 and the Gauss-Bonnet correction have the same role.

Inserting the condition $T_5^0 = 0$ in Eq. (26), it reduces to $\dot{\rho} + 3H\rho(1+\omega) = 0$. This equation has well known solution $\rho = \rho_0 a^{-3(1+\omega)}$, where ρ_0 is the present matter density of the universe and we have omitted the “o” subscript from the scale factor on the brane for simplicity. Then, consider a general ansatz $T_5^5 = Da^\nu$ for the bulk pressure [18], where D and ν are two arbitrary constants, one can easily check that Eq. (31) has a solution of the form

$$\chi = Ca^{-4} + Ba^\nu, \quad (32)$$

where C is a constant usually referred to as dark radiation term and $B \equiv D(\nu + 2)/(3\nu + 12)$. Finally, inserting ρ and χ into Eq. (30), we can rewrite it in the standard form

$$H^2 = \frac{8\pi G_N}{3}(\rho + \rho_{\text{eff}}), \quad (33)$$

where $G_N = 3\gamma/4\pi$ is the 4-dimensional Newtonian constant and ρ_{eff} represents the effective dark energy density on the brane

$$\rho_{\text{eff}} = \frac{1}{2\gamma} (Ca^{-4} + Aa^\nu) - \frac{\alpha}{36\gamma} \left[2(Ca^{-4} + Aa^\nu)(\sigma^2 + 18(Ca^{-4} + Aa^\nu)) - \frac{\sigma^4}{108} \right], \quad (34)$$

where $A \equiv -2D/(3\nu + 12)$. The equation of state parameter of the effective dark energy on the brane can be defined by [25]

$$w_{\text{eff}} = -1 - \frac{1}{3} \frac{d \ln \delta H^2}{d \ln a}, \quad (35)$$

where $\delta H^2 = (H^2/H_0^2) - \Omega_m a^{-3}$ accounts for terms in the Friedmann equation except the brane matter with equation of state $w_m = 0$. Now, if we use the redshift parameter $1 + z = a^{-1}$ as our variable, we can easily show that

$$\begin{aligned} \omega_{\text{eff}}(z) = & -1 + \frac{1}{3} \left(4C(1+z)^4 - A\nu(1+z)^{-\nu} \right) \left[1 - \frac{\alpha}{9} \left(36A(1+z)^{-\nu} + 36C(1+z)^4 + \sigma^2 \right) \right] \\ & \times \left\{ C(1+z)^4 + A(1+z)^{-\nu} - \frac{\alpha}{18} \left[2 \left(A(1+z)^{-\nu} + C(1+z)^4 \right) \right. \right. \\ & \left. \left. \times \left(\sigma^2 + 18A(1+z)^{-\nu} + 18C(1+z)^4 \right) - \frac{\sigma^4}{108} \right] \right\}^{-1}. \end{aligned} \quad (36)$$

The corresponding late time deceleration parameter can be written

$$q(z) \equiv -\frac{1}{H^2} \frac{\ddot{a}}{a} = \frac{1}{2} [\Omega_m + (1 - \Omega_m)(1 + 3\omega_{\text{eff}}(z))], \quad (37)$$

where $\Omega_m = \Omega_{m0} (1+z)^3$ is all part of the matter on the braneworld and we take its present value as $\Omega_{m0} = 0.28 \pm 0.02$. In the rest of the paper, we will obtain constraints on the parameters such as C , A , ν , α and σ in our model. Indeed, we want to show that under what parameter space constraints our model can describe the accelerated expansion of the universe with the equation of state of the effective dark energy ω_{eff} crossing -1 , as suggested by observations.

A. Special case with $\alpha = 0$

Let us begin with the special case, in which the Gauss-Bonnet coupling constant α is equal to zero. In this case we have the usual Randall-Sundrum II braneworld model and Eq. (36) reduces to

$$w_{\text{eff}}(z) = -1 + \frac{1}{3} \left(\frac{4C(1+z)^{\nu+4} - A\nu}{C(1+z)^{\nu+4} + A} \right). \quad (38)$$

Therefore, we are left with three parameters C , A , ν , and two of them are independent. Requiring that at the present moment $w_{\text{eff}}(z=0) = -1.06$ and w crossed -1 around $z = 0.2$ as indicated by extensive analysis of observational data [9], we can obtain

$$C = 0.039A, \quad \nu = 0.34, \quad A = A. \quad (39)$$

For these value of parameters and $\Omega_{m0} = 0.28$, from Eq. (37) we have $q(z=0) = -0.64$ and in addition $q(z)$ crosses 0 around $z = 0.33$ which is in good agreement with recent observational data [1][24]. In figure 1 we plot $w_{\text{eff}}(z)$ and $q(z)$ for the above value of the parameters versus redshift parameter z .

B. General case with $\alpha \neq 0$

Next, we consider the general Gauss-Bonnet braneworld with bulk matter content. In this case we have five parameters only four of which are independent. Considering that the value of σ does not affect the general profile of our model and further according to the Randall-Sundrum fine-tuning relation it should be small, we first fix $\sigma = 10^{-3}$. Thus, we have now four parameters and among them three are independent. Numerical calculations show that the functions w_{eff} and q are well behaved for $z \geq 0$, provided that $0 < \nu \leq 0.34$. Employing the present value of the equation of state parameter of dark energy $\omega_{\text{eff}}(z=0) = -1.06$ and the moment it crossed -1 , namely $\omega_{\text{eff}}(z=0.2) = -1$, we get

$$\alpha = \alpha(A, \nu), \quad C = 0.12 A \nu (1.2)^{-\nu}, \quad A = A. \quad (40)$$

If we impose the condition $0 < \alpha < 1$ which was used in deriving our equations, we can get constraint on the free parameter A . In numerical calculations we find that for $0 < \nu \leq 0.17$ we should have $A > 17.97$, while for $0.18 \leq \nu \leq 0.34$ we should have $A < -5.23$ to satisfy the condition on α . In figures 2 and 3 we plot $w_{\text{eff}}(z)$ and $q(z)$ for $0 < \nu \leq 0.17$ versus redshift parameter z for different value of the Gauss-Bonnet coupling constant α . From these figures we observe that at large z , the $w_{\text{eff}}(z)$ increases with the increase of α , while $q(z)$ decreases with the increase of α . This qualitative behavior is quite opposite when $0.18 \leq \nu \leq 0.34$ as one can see from figures 4 and 5. Finally we plot in figures 6-9 the parameter space for the functions $\alpha = \alpha(A, \nu)$ and $C = C(A, \nu)$. We find that in the case $0 < \nu \leq 0.17$, α and C increase with the increase of ν while, in contrast, for $0.18 \leq \nu \leq 0.34$, α and C decrease with the increase of ν .

IV. CONCLUSIONS AND DISCUSSIONS

In this work we have generalized the Randall-Sundrum II braneworld with both bulk matter content and bulk-brane energy exchange by adding the Gauss-Bonnet curvature correction term in the bulk action. We have investigated the effects of the bulk content in the general Gauss-Bonnet braneworld on the evolution of the universe and found that although the effect of the Gauss-Bonnet correction term in the late time universe is small, it still plays an important role in the universe evolution.

In contrast to the previous models ([15, 16, 17, 18]), in our study we kept the energy momentum conservation law on the brane as usual and found that the combination of the dark radiation term and the matter content of the bulk can play the role of the dark energy on the brane and influence the evolution of the universe. By suitably choosing parameter space in our model, we can describe the super-acceleration of our universe with the behavior of the effective dark energy equation of state in agreement with observations. In [22], it was argued in a Gauss-Bonnet brane world with induced gravity that the Gauss-Bonnet term and the mass parameter in the bulk play a crucial role in the evolution of the universe. Here in our general model, we confirmed their argument. It is easy to see from Eqs.(25)-(27) that the Gauss-Bonnet correction influences the dynamics of our universe, especially in the early universe with high energy scale. Phenomenon on the Gauss-Bonnet role has been disclosed in Figs.2-5. We observed that although the Gauss-Bonnet effect is not clear at the present moment, it influences the universe evolution in the past and was more important in the earlier period.

In this work we just restricted our numerical fitting to limited observational data. Giving the wide range of cosmological data available, in the future we expect to further constrain our model parameter space and test the viability of our model.

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